University of Victoria | Graduate Studies

Notice of the Final Oral Examination for the Degree of Master of Science

of

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BSc (University of British Columbia, 2013)

"Refined Inertias Related to Biological Systems and to the Petersen Graph"

Department of Mathematics and Statistics

Tuesday, August 18, 2015
1:30 P.M.
Clearihue Building
Room B215

Supervisory Committee:
Dr. Pauline van den Driessche, Department of Mathematics and Statistics, University of Victoria (Co-Supervisor)
Dr. Dale D. Olesky, Department of Computer Science, UVic (Co-Supervisor)

External Examiner:
Dr. Gary MacGillivray, Department of Mathematics and Statistics, UVic

Chair of Oral Examination:
Dr. Eric Sager, Department of History, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

Many models in the physical and life sciences formulated as dynamical systems have a positive steady state, with the local behavior of this steady state determined by the eigenvalues of its Jacobian matrix. The first part of this thesis is concerned with analyzing the linear stability of the steady state by using sign patterns, which are matrices with entries from the set f+;; 0g. The linear stability is related to the allowed refined inertias of the sign pattern of the Jacobian matrix of the system, where the refined inertia of a matrix is a 4-tuple (n+; n□; nz; 2np) with n+ (n□) equal to the number of eigenvalues with positive (negative) real part, nz equal to the number of zero eigenvalues, and 2np equal to the number of nonzero pure imaginary eigenvalues. This type of analysis is useful when the parameters of the model are of known sign but unknown magnitude. The usefulness of sign pattern analysis is illustrated with several biological examples, including biochemical reaction networks, Predator-prey models, and an infectious disease model. The refined inertias allowed by sign patterns with specific digraph structures have been studied, for example, for tree sign patterns. In the second part of this thesis, such results on refined inertias are extended by considering sign and zero-nonzero patterns with digraphs isomorphic to strongly connected orientations of the Petersen graph.